Chaos synchronization regimes in multiple-time-delay semiconductor lasers

E. M. Shahverdiev[1,2] and K. A. Shore[1]
[1]*School of Electronics, University of Wales, Bangor, Dean Street, Bangor LL57 1UT, Wales, United Kingdom*
[2]*Institute of Physics, H. Javid Avenue, 33, Baku AZ1143, Azerbaijan*

(Received 25 July 2007; revised manuscript received 11 February 2008; published 7 May 2008)

This paper provides a report of chaos synchronization regimes which may be accessed in both unidirectionally and bidirectionally coupled multiple-time-delay semiconductor lasers. We demonstrate that in bidirectionally coupled multiple-time-delay lasers additional feedback can considerably enhance synchronization quality.

DOI: 10.1103/PhysRevE.77.057201 PACS number(s): 05.45.Xt, 05.45.Vx, 42.55.Px, 42.65.Sf

Recently delay differential equations (DDEs) have attracted a great deal of attention in the field of nonlinear dynamics [1]. In comparison with a single-time-delay system DDEs with multiple time delays provide more realistic models of interacting complex systems. From the applications viewpoint additional time delays offer the opportunity, e.g., to stabilize the output of a nonlinear system [2,3]. Recently it was shown that nonlinear systems with two delay times offer a higher complexity of dynamics than achievable in more conventional single delay time systems [4]. The high complexity of such multiple-time-delay systems can provide a new architecture for enhancing message security in chaos based encryption systems [4].

External cavity semiconductor lasers are an integral part of high speed chaos-based communication systems [5]. In many practical applications it may occur that such lasers are subject to more than one optical reflection (whether deliberate or unwanted) and thus may represent a system with multiple time delays. Laser diodes with several external cavities could offer higher security for such communication systems. For message decoding in such a system one must be able to synchronize the transmitter and receiver lasers. In this Brief Report we offer the first analysis of chaos synchronization regimes in both unidirectionally and bidirectionally coupled multiple time-delay semiconductor laser systems. This work provides the basis for experimental investigations directed at the deployment of such lasers in secure high-speed information processing.

To model unidirectionally coupled external cavity semiconductor lasers with double time delays we use the paradigm Lang-Kobayashi equations [6]. For the slowly varying complex amplitude E and the electron population N. The dynamics of the master laser is governed by the following system:

$$\frac{dE_1(t)}{dt} = \frac{1}{2}\{-i\alpha G_N[N_1(t)-N_{\rm sol}] + [G(N_1)-\Gamma]\}E_1(t) + k_1 E_1(t-\tau_1)e^{i\omega\tau_1} + k_2 E_1(t-\tau_2)e^{i\omega\tau_2} + \beta_1\xi_1(t), \tag{1}$$

$$\frac{dN_1(t)}{dt} = pJ_{\rm th} - \gamma N_1(t) - G(N_1)|E_1(t)|^2. \tag{2}$$

The behavior of the slave laser is described by the following set of equations:

$$\frac{dE_2(t)}{dt} = \frac{1}{2}\{-i\alpha G_N[N_2(t)-N_{\rm sol}] + [G(N_2)-\Gamma]\}E_2(t) + k_3 E_2(t-\tau_1)e^{i\omega\tau_1} + k_4 E_2(t-\tau_2)e^{i\omega\tau_2} + \sigma E_1(t-\tau_3)e^{i\omega\tau_3} + \beta_2\xi_2(t), \tag{3}$$

$$\frac{dN_2(t)}{dt} = pJ_{\rm th} - \gamma N_2(t) - G(N_2)|E_2(t)|^2, \tag{4}$$

where $N_{\rm sol}$ is the solitary laser carrier number; G_N is the differential optical gain; $\tau_{1,2}$ is the external cavities roundtrip time; α is the linewidth enhancement factor; γ is the carrier decay rate; Γ is the cavity decay rate; p is the pump current relative to the threshold value of the solitary laser $J_{\rm th}=\gamma N_{\rm sol}$; and ω is the angular frequency of the solitary laser. The value of ω is of no importance for the calculations and below in numerical simulations we take $\lambda=\frac{2\pi c}{\omega}=1550$ nm (widely used in long-haul fiber-optic communication systems); $k_{1,2}$ and $k_{3,4}$ are the feedback rates for the master and slave lasers, respectively; σ is the coupling strength between lasers; τ_3 is the time of flight between the lasers. $\xi_{1,2}(t)$ are independent complex Gaussian white noises of zero mean and $\beta_{1,2}$ measures the noise intensity. Below $x_\tau \equiv x(t-\tau)$ and synchronous solutions between real electric field amplitudes $|E|$ are presented.

The Lang-Kobayashi model does not include multiple roundtrip reflections in the external cavity, but nevertheless the model is valid for feedback coefficients as high as 30 ns^{-1} [7]. It is noted that for $k_1=k_3=0$ (or $k_2=k_4=0$) we obtain the case of unidirectionally coupled single cavity laser diodes [6]. Comparing Eqs. (1) and (2) and Eqs. (3) and (4) (noise is neglected) one finds that a synchronous solution $|E_2|=|E_{1,\tau_3-\tau_1}|$ exists if $k_1=k_3+\sigma, k_2=k_4$. One can notice that for $\tau_3=\tau_1$ synchronization is complete; for $\tau_3>\tau_1$ lag synchronization can take place; for $\tau_3<\tau_1$ anticipation synchronization is expected [8]. Analogously the synchronous solution $|E_2|=|E_{1,\tau_3-\tau_2}|$ exists if $k_2=k_4+\sigma$, $k_1=k_3$. Notice that with an additional external cavity the number of possible synchronization manifolds is doubled.

Most chaos based communication techniques use synchronization in the unidirectional master-slave system. As a two-way transmission of signals require bidirectional coupling, in this paper we also consider chaos synchronization between the mutually coupled laser diodes with double time delays. For the symmetrical bidirectional coupling one must

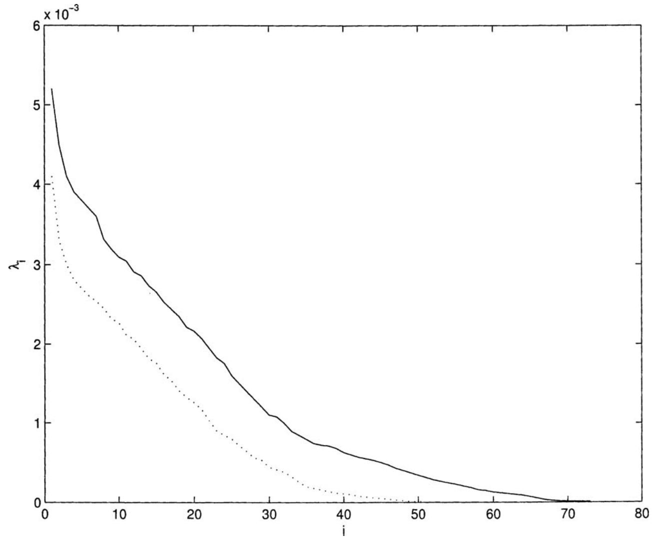

FIG. 1. Numerical simulation of the Lang-Kobayashi model, Eqs. (1) and (2) for a single delay ($k_1=30\ \text{ns}^{-1}$, $\tau_1=10$ ns) and double time delays ($k_1=30\ \text{ns}^{-1}$, $\tau_1=10$ ns and $k_2=25\ \text{ns}^{-1}$, $\tau_2=7$ ns): Spectrum of the positive Lyapunov exponents for the single time delay (dotted line) and double time delays (solid line).

add the term $\sigma E_2(t-\tau_3)e^{i\omega\tau_3}$ to the right-hand side of Eq. (1). Now comparing the system of equations (1) and (2) modified in such a way with the system of equations (3) and (4) we establish that complete synchronization $|E_1|=|E_2|$ is possible under the conditions $k_1=k_3$, $k_2=k_4$.

We note that the existence conditions are necessary for synchronization, but from the application viewpoint the stability of the synchronization regimes is of paramount importance. Due to the high complexity of the model under study to identify stable synchronous regimes, recourse must be made to numerical modeling.

In numerical simulations we use the following values for the parameters: $N_{\text{sol}}=1.7\times10^8$, $G_n=2.14\times10^4\ \text{s}^{-1}$, $\alpha=5$, $\gamma=0.9\ \text{ns}^{-1}$, $\Gamma=0.36\ \text{ps}^{-1}$, $p=1.02$, see, e.g., Ref. [20] in [6].

First we demonstrate that the multiple-time-delay laser system can offer higher complexity than a single-time-delay laser. For the computation of the Lyapunov exponents we have applied the method advanced by Farmer [9], integrating the system of equations (1) and (2) with a Euler method. Figure 1 shows the spectrum of positive Lyapunov exponents for the single (dotted line) and double (solid line) delay times for $k_1=30\ \text{ns}^{-1}$, $\tau_1=10$ ns and $k_2=25\ \text{ns}^{-1}$, $\tau_2=7$ ns. The computation of the Kolmogorov-Sinai (KS) entropy—the sum of all the positive Lyapunov exponents [7]—reveals that for a given set of parameters a double-time-delay system shows more unpredictability than the single-time-delay system, $h_{\text{KS}}(\text{double time delay})\approx0.078>h_{\text{KS}}(\text{single time delay})\approx0.059$. We have also computed the information dimension from the Kaplan-Yorke formula $d_{\text{KY}}=j+|\lambda_{j+1}|^{-1}\Sigma_{i=1}^{i=j}\lambda_i$, where the integer j, that represents the number of degrees of freedom, meets the conditions $\Sigma_{i=1}^{i=j}\lambda_i>0$ and $\Sigma_{i=1}^{i=j+1}\lambda_i<0$ when the Lyapunov exponents are ordered by their magnitude from positive to negative values [7], $d_{\text{KY}}(\text{single time delay})=103.5<d_{\text{KY}}(\text{double time delay})=145.7$. We note the important difference between the single- and double-time-delay cases: In the single-time-delay case with increase of the delay time the number of positive Lyapunov exponents increases, but their magnitude decreases; in the double-time-delay case both the number and magnitude of positive Lyapunov exponents increase with increased delay time. This means that double-time-delay systems can offer more complexity and unpredictability than single-time-delay systems.

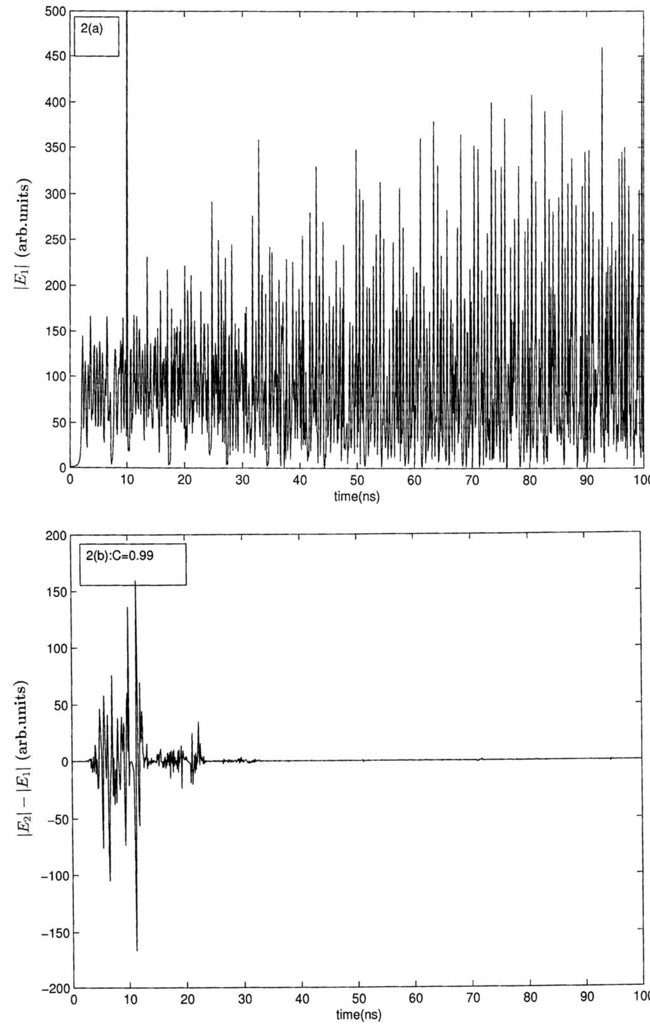

FIG. 2. Numerical simulation of the Lang-Kobayashi model, Eqs. (1)–(4), for $k_1=30\ \text{ns}^{-1}$, $k_2=25\ \text{ns}^{-1}=k_4$, $k_3=1\ \text{ns}^{-1}$, $\sigma=29\ \text{ns}^{-1}$, $\tau_1=10\ \text{ns}=\tau_3$, $\tau_2=7$ ns. Complete synchronization between $|E_1(t)|$ and $|E_2(t)|$: (a) time series of $|E_1|$; (b) synchronization error $|E_2|-|E_1|$ dynamics. C is the correlation coefficient between $|E_2|$ and $|E_1|$.

Numerical simulations of synchronization regimes were conducted using the DDE23 program in Matlab 7. First we consider the case of complete synchronization between unidirectionally coupled master (transmitter) and slave (receiver) laser systems. According to our analytical findings, synchronization $|E_1|=|E_2|$ could occur if $k_1=k_3+\sigma$, $k_2=k_4$ for $\tau_3=\tau_1$ or for conditions $k_1=k_3$, $k_2=k_4+\sigma$, and $\tau_3=\tau_1$. Figure 2 demonstrates complete synchronization for $k_1=30\ \text{ns}^{-1}$, $k_2=25\ \text{ns}^{-1}=k_4$, $k_3=1\ \text{ns}^{-1}$, $\sigma=29\ \text{ns}^{-1}$, $\tau_1=10\ \text{ns}=\tau_3$, $\tau_2=7$ ns. Figure 3 shows lag synchronization $|E_2|=|E_{1,\tau_3-\tau_1}|$ ($\tau_3>\tau_1$) for $k_1=30\ \text{ns}^{-1}$, $k_2=5\ \text{ns}^{-1}=k_4$, $k_3=2$

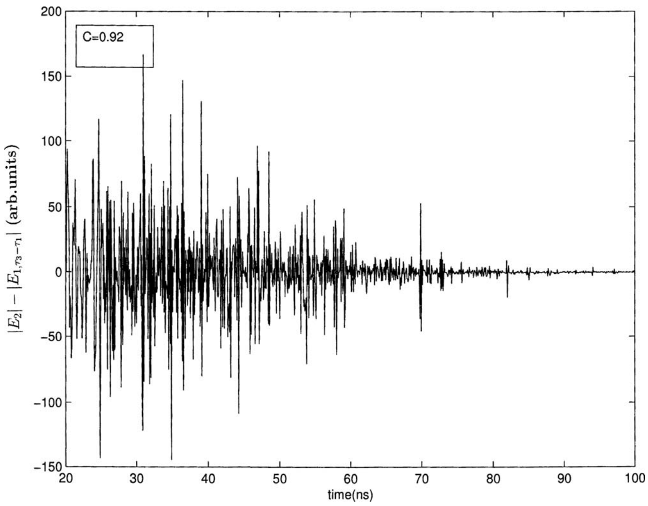

FIG. 3. Numerical simulation: Lag synchronization between $|E_1(t)|$ and $|E_2(t)|$, $k_1=30$ ns^{-1}, $k_2=5$ ns$^{-1}=k_4$, $k_3=2$ ns^{-1}, $\sigma=28$ ns^{-1}, $\tau_1=15$ ns, $\tau_2=12$ ns, $\tau_3=23$ ns, synchronization error $|E_2|-|E_{1,\tau_3-\tau_1}|$ dynamics. C is the correlation coefficient between $|E_2|$ and $|E_{1,\tau_3-\tau_1}|$.

ns^{-1}, $\sigma=28$ ns^{-1}, $\tau_1=15$ ns, $\tau_2=12$ ns, $\tau_3=23$ ns with feedback and coupling strengths satisfying the existence conditions derived above. The anticipating synchronization manifold $|E_2|=|E_{1,\tau_3-\tau_2}|$ (Fig. 4, $\tau_3<\tau_2$) is also observed in full accordance with the analytic results established above. Note that synchronization manifold $|E_2|=|E_{1,\tau_3-\tau_2}|$ is equivalent to $|E_1|=|E_{2,\tau_2-\tau_3}|$. Numerical simulations of bidirectionally coupled laser diodes with double time delays are presented in Fig. 5.

We note that complete synchronization between mutually coupled single-time-delay laser systems was studied in recent work [10]. The authors of [10] have established that when both lasers are subject to approximately the same feedback and coupling strengths, high-quality identical synchronization was achievable. Most importantly in [10] it was found that a necessary condition for high-quality identical synchronization is that the feedback round-trip of both lasers be equal to the coupling length; in other words, the feedback delay time should be equal to the coupling time between the lasers. In this paper we investigate the dependence of the identical synchronization quality (in terms of cross-correlation coefficients between the synchronized lasers) on the ratio between the feedback and coupling delay times. By these means it is shown that additional time delays can enhance the synchronization quality when the feedback and

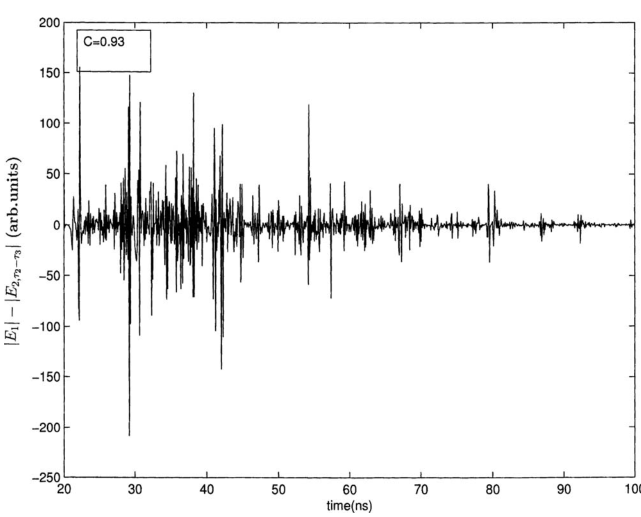

FIG. 4. Numerical simulation: Anticipating synchronization between $|E_1(t)|$ and $|E_2(t)|$: $|E_1|=|E_{2,\tau_2-\tau_3}|$ for $k_1=4$ ns$^{-1}=k_3$, $k_2=29$ ns^{-1}, $k_4=1$ ns^{-1}, $\sigma=28$ ns^{-1}, $\tau_1=25$ ns, $\tau_2=20$ ns, $\tau_3=15$ ns, synchronization error $|E_1|-|E_{2,\tau_2-\tau_3}|$ dynamics. C is the correlation coefficient between $|E_1|$ and $|E_{2,\tau_2-\tau_3}|$.

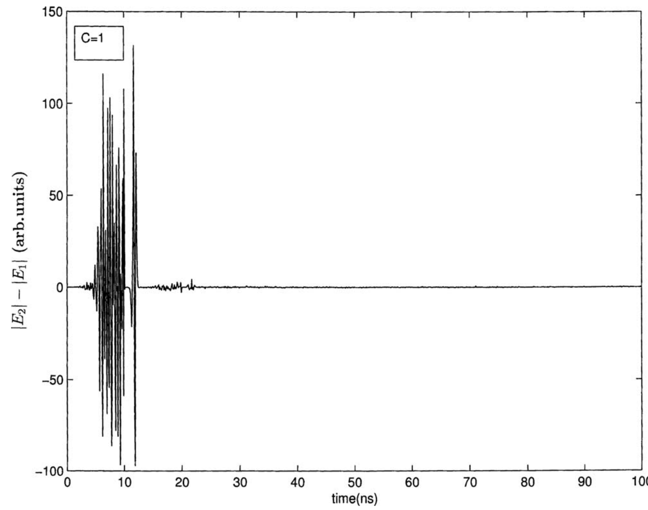

FIG. 5. Numerical simulation of the bidirectionally coupled Lang-Kobayashi model: Complete synchronization between $|E_1(t)|$ and $|E_2(t)|$ for $k_1=k_3=25$ ns^{-1}, $k_2=k_4=13$ ns^{-1}, $\sigma=29$ ns^{-1}, $\tau_1=40$ ns, $\tau_2=20$ ns, $\tau_3=39$ ns, synchronization error $|E_2|-|E_1|$ dynamics. C is the correlation coefficient between $|E_2|$ and $|E_1|$.

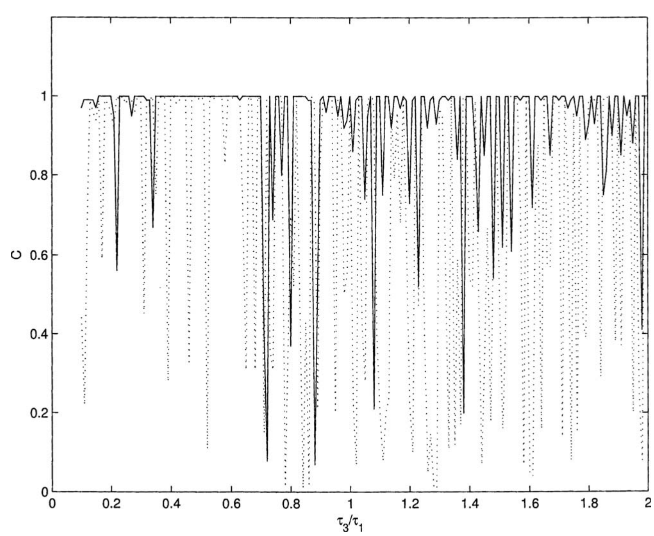

FIG. 6. Numerical simulation of the bidirectionally coupled Lang-Kobayashi model for $k_1=k_3=30$ ns^{-1}, $k_2=k_4=10$ ns^{-1}, $\sigma=20$ ns^{-1}, $\tau_1=10$ ns, $\tau_2=50$ ns: Dependence of the cross-correlation coefficient C on the ratio of the coupling delay time τ_3 to the feedback delay time τ_1; dotted line, single-time-delay systems; solid line, double-time-delay systems.

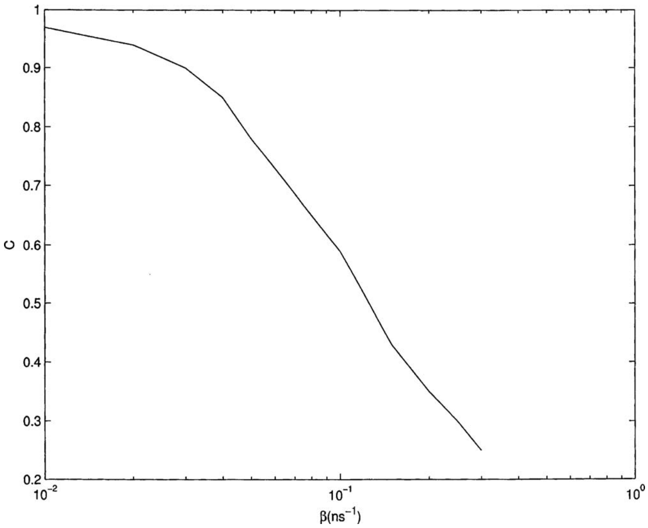

FIG. 7. The correlation coefficient C between $|E_2|$ and $|E_1|$ for different noise intensities $\beta = \beta_1 = \beta_2$ for parameters as in Fig. 2. Note the logarithmic scale on the abscissa axis.

coupling delay times for single-time-delay systems are very different. Figure 6 shows a considerable improvement in synchronization quality for systems with double time delays when feedback and coupling delay times for the single feedback systems differ. The result emphasizes the stabilizing role of additional time delays [3].

We have also studied the influence of noise on the synchronization quality. We have used a fourth-order Runge-Kutta method to integrate Eqs. (1)–(4) with noise terms [11]. Figure 7 demonstrates the effect of the noise intensity on the correlation coefficient for complete synchronization for parameters as in Fig. 2; synchronization is robust to small noise intensity ($\beta_1 = \beta_2 = \beta < 0.05$ ns^{-1}), but noise intensity at the level of 0.5 ns^{-1} results in the loss of synchronization.

In conclusion we have presented a report of the chaos synchronization regimes in both unidirectionally and bidirectionally coupled multiple-time-delay semiconductor lasers. We have demonstrated that in bidirectionally coupled multiple-time-delay lasers additional feedback can considerably enhance synchronization quality. We have also established the multiplicity of synchronization regimes for multiple-time-delay systems. Such multiplicity of synchronization manifolds could provide a certain degree of flexibility in practical applications. These results also provide the basis for the use of semiconductor laser diodes in chaos-based secure high-speed information processing.

This research was supported by a Marie Curie Action within the 6th European Community Framework Programme Contract.

[1] M. C. Mackey and L. Glass, Science **197**, 287 (1977); R. Lang and K. Kobayashi, IEEE J. Quantum Electron. **16**, 347 (1980); E. Tziperman *et al.*, Science **264**, 72 (1994); J. K. Hale and S. M. V. Lunel, *Introduction to Functional Differential Equations* (Springer, New York, 1993); C. R. Mirasso *et al.*, C. R. Phys. **5**, 613 (2004).

[2] Y. Liu and J. Ohtsubo, IEEE J. Quantum Electron. **33**, 1706 (1997).

[3] A. Ahlborn and U. Parlitz, Phys. Rev. Lett. **93**, 264101 (2004).

[4] M. W. Lee, L. Larger, V. Udaltsov, E. Genin, and J. P. Goedgebuer, Opt. Lett. **29**, 325 (2004).

[5] D. M. Kane and K. A. Shore, *Unlocking Dynamical Diversity: Optical Feedback Effects on Semiconductor Lasers* (Wiley, New York, 2005).

[6] I. Wedekind and U. Parlitz, Phys. Rev. E **66**, 026218 (2002).

[7] R. Vicente, J. Dauden, P. Colet, and R. Toral, IEEE J. Quantum Electron. **41**, 541 (2005).

[8] C. Masoller, Phys. Rev. Lett. **86**, 2782 (2001).

[9] J. D. Farmer, Physica D **4**, 366 (1982).

[10] N. Gross *et al.*, Opt. Commun. **267**, 464 (2006).

[11] P. E. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations* (Springer, New York, 2000).